Appendix 2.5
Derivation of No Treat-Test and Test-Treat Probability Thresholds

B = cost (regret) of failing to treat a $D+$ individual
C = cost (regret) of treating a $D-$ individual unnecessarily
T = cost of test
P = probability of $D+$

$p[-|D+]$ = probability of negative test given $D+$ = 1– sensitivity
$p[+|D-]$ = probability of positive test given $D-$ = 1– specificity

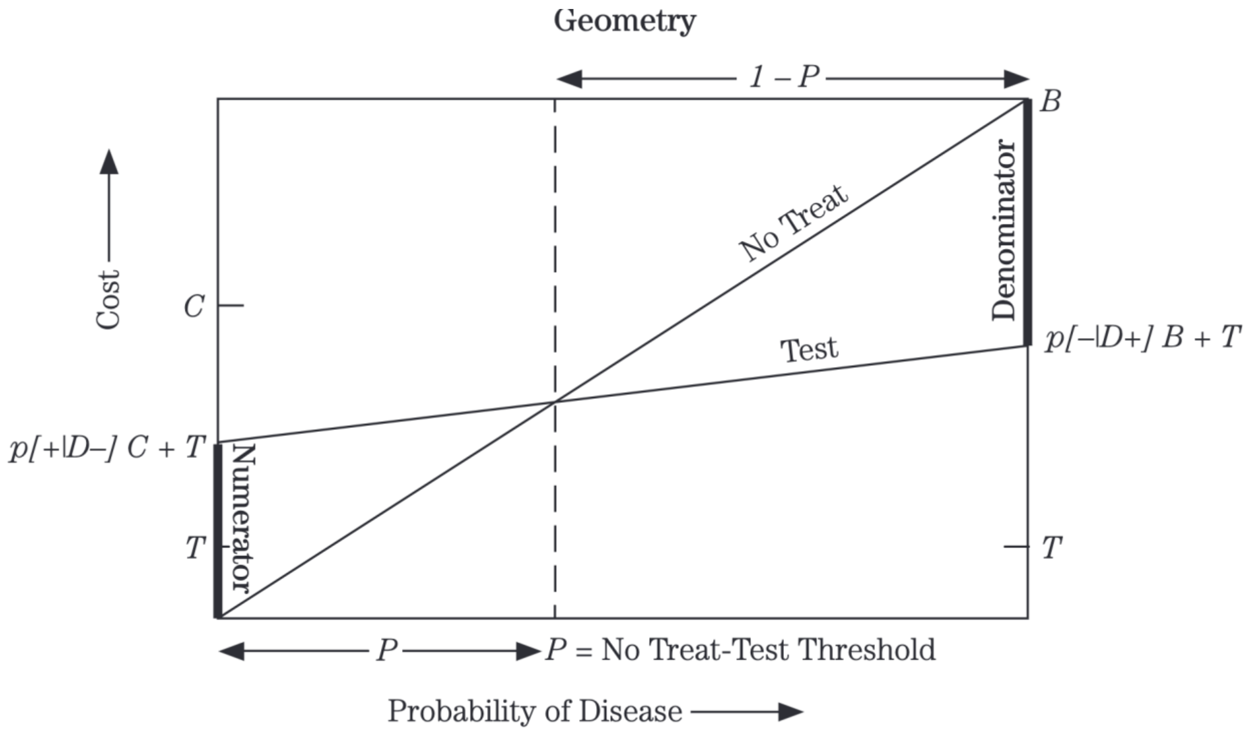
Expected Cost (Regret) of No Treat strategy: $(P)B$

Expected Cost (Regret) of Treat Strategy: $(1-P)C$

Expected Cost of Test Strategy:
$P\,(p[-|D+])\,B + (1-P)\,(p[+|D-])\,C + T$

Reminder about odds and probability:
Convert odds to probability by replacing the denominator with the sum of the numerator and the denominator. If threshold odds are C/B, then threshold probability is C/(B+C).

No Treat-Test Threshold
Geometry



Probability of Disease ⟶

Convince yourself that the ratio of the line labeled “Numerator” to the line labeled “Denominator” is equal to the threshold odds P/(1–P)

$$P / (1-P) = \frac{(p[+|D-])C + T}{B - (p[-|D+])B - T}$$

$$= \frac{(p[+|D-]\ C + T}{B\ (1 - p[-|D+]) - T}$$

Substitute $p[+|D+]$ *for* $1 - p[-|D+]$

$$= \frac{(p[+|D-])\ C + T}{(p[+|D+])\ B - T}$$

= No Treat-Test Threshold Odds
(add numerator to denominator for probability)

$$= \frac{(p[+|D-])\ C + T}{(p[+|D+])\ B + (p[+|D-])\ C}$$ = No Treat-Test Threshold Probability

No Treat-Test Threshold
Algebra

No Treat-Test threshold is where the expected cost of the "No Treat" strategy equals the expected cost of the "Test" strategy.

$$(P)(B) = P\,(p[-|D+])\,B + (1-P)\,(p[+|D-])\,C + T$$

Substitute $(P)(T) + (1-P)T$ for T

$$(P)(B) = P\,(p[-|D+])\,B + (P)(T) + (1-P)\,(p[+|D-])\,C + (1-P)\,T$$

$$(P)(B) = \underbrace{P\,(p[-|D+]\,B + T)}_{\text{subtract this}} + (1-P)\,(p[+|D-]\,C + T)$$

$$(P)(B) - P\,(p[-|D+]\,B + T) = (1-P)\,(p[+|D-]\,C + T)$$

$$(P)[(B)\,(1 - p[-|D+]\,) - T] = (1-P)\,(p[+|D-]\,C + T)$$

Substitute $p[+|D+]$ for $1-p[-|D+]$

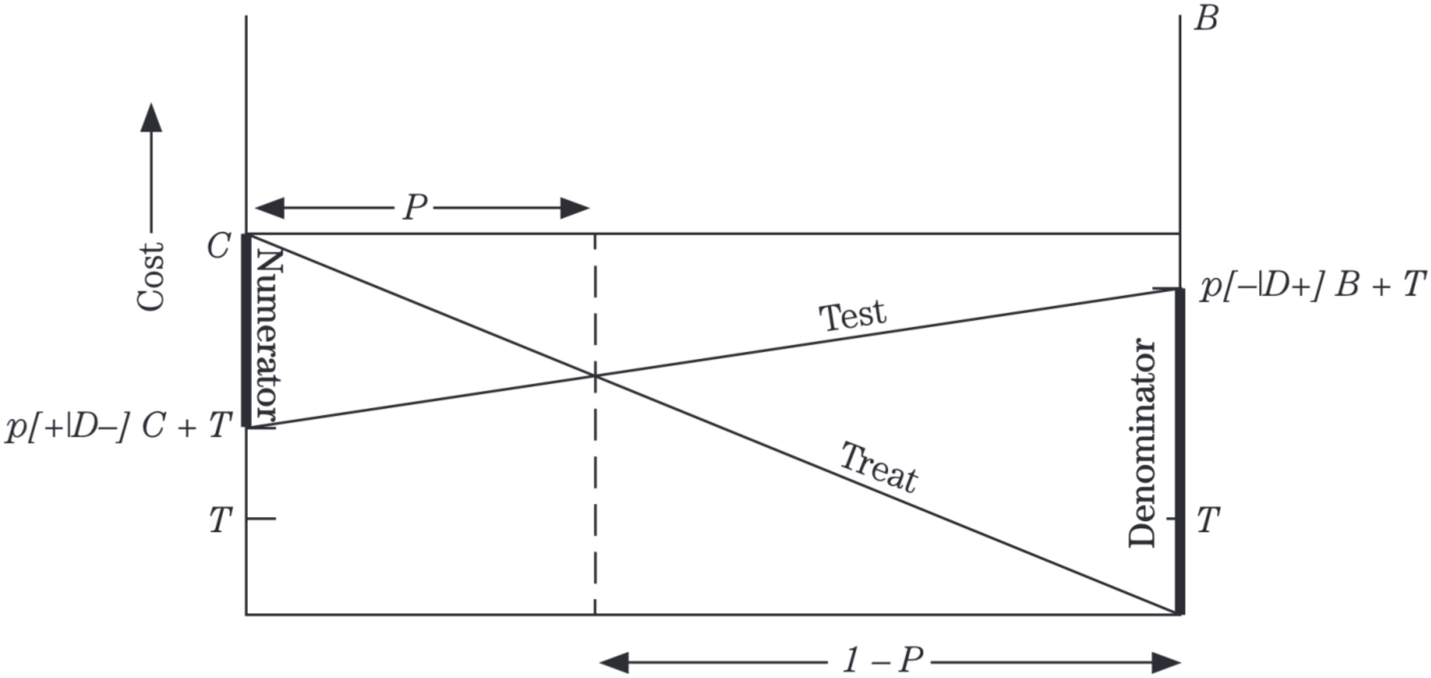
$$(P)\,[p[+|D+](B) - T] = (1-P)\,(p[+|D-]\,C + T)$$

$$\frac{P}{(1-P)} = \frac{p[+|D-]\,C + T}{p[+|D+]\,B - T}$$

This is threshold odds. To get threshold probability add the numerator to the denominator.

$$P = \frac{(p[+|D-])\,C + T}{(p[+|D+])\,B + (p[+|D-])\,C} = \text{No Treat-Test Threshold Probability}$$

Test-Treat Threshold Geometry



Convince yourself that

$$P \,/\, (1-P) = \frac{C - (p[+\,|\,D-])\ C + T}{(p[-\,|\,D+])\ B + T}$$

$$= \frac{C\ (1 - p[+\,|\,D-] - T}{p[-\,|\,D+]\ B + T}$$

Substitute $p[-\,|\,D-]$ for $1 - p[+\,|\,D-]$

$$= \frac{C\ (p[-\,|\,D-]) - T}{(p[-\,|\,D+])\ B + T}$$

= Test-Treat Threshold Odds
(add numerator to denominator for probability)

$$P = \frac{(p[-\,|\,D-])\ C - T}{(p[-\,|\,D+])\ B + (p[-\,|\,D-])\ C}$$ = Test-Treat Threshold Probability

Test-Treat Threshold
Algebra

The Test-Treat strategy is where the expected cost of the "Test" strategy equals the expected cost of the "Treat" strategy.

$$P\,(p[-|D+])\,B + (1-P)\,(p[+|D-])\,C + T = (1-P)\,C$$

Substitute $(P)(T) + (1-P)T$ for T

$$P\,(p[-|D+])\,B + (P)(T) + (1-P)\,(p[+|D-])\,C + (1-P)\,T = (1-P)\,C$$

$$P\,(p[-|D+]\,B + T) + (1-P)\,(p[+|D-]\,C + T) = (1-P)\,C$$

Subtract this

$$P\,(p[-|D+]\,B + T) = (1-P)\,C - (1-P)\,(p[+|D-]\,C + T)$$

Rearrange

$$P\,(p[-|D+]\,B + T) = (1-P)\,(1 - p[+|D-]) - (1 - P)\,T$$

Substitute $p[-|D-]$ for $1 - p[+|D-]$

$$P\,(p[-|D+]\,B + T) = (1-P)\,(p[-|D-]\,C - T)$$

$$\frac{P}{(1-P)} = \frac{p[-|D-]\,C - T}{p[-|D+]\,B + T}$$

This is threshold odds. To get threshold probability add the numerator to the denominator.

$$P = \frac{p[-|D-]\,C - T}{p[-|D+]\,B + p[-|D-]\,C} = \text{Test-Treat Threshold Probability}$$