Ch03.71.A. Septic Arthritis of the Knee and WBC Count in the Joint Fluid
Septic arthritis is a bacterial infection in a joint. Patients with septic arthritis of the knee present with a painful, swollen, warm knee, but other conditions such as gout or pseudogout can cause a similar presentation. One test for septic arthritis is to insert a needle into the joint space, withdraw fluid, and send it to the lab for a white blood cell (WBC) count. Septic arthritis tends to cause higher WBC counts than the non-septic arthritis conditions.

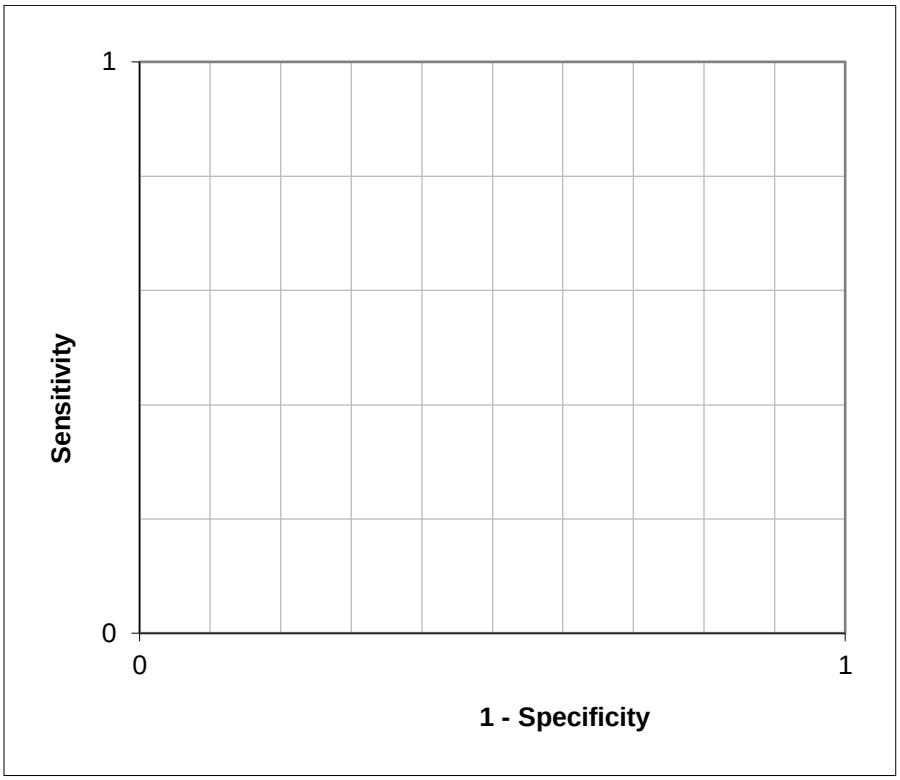
You study 15 consecutive patients who presented to the emergency department with a painful, swollen, warm knee, and who had joint fluid WBC counts. On all 15 patients, a final diagnosis was established by an independent, valid gold standard. Five had septic arthritis, ten had something else. Here are the joint fluid WBC counts:

Septic Arthritis	Not Septic Arthritis
30	0
37	6
64	7
112	8
128	12
	12
	23
	37
	48
	71

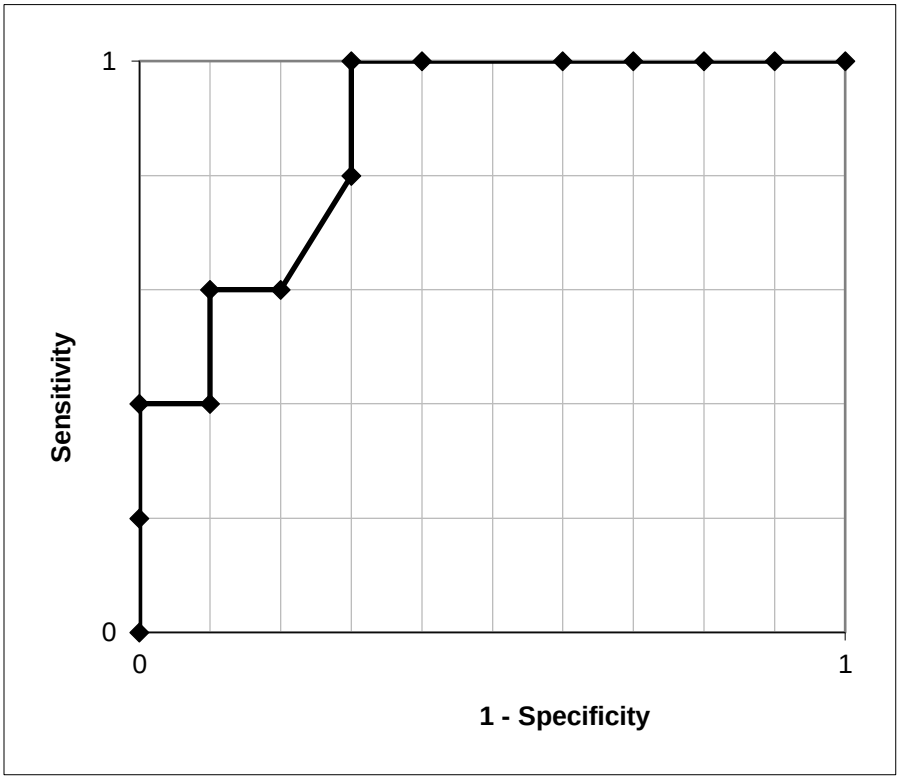
We are going to ask you to draw the ROC curve, so we are doing you the favor of sorting the test results from most abnormal to least abnormal:

Septic Arthritis	No Septic Arthritis
128	
112	
	71
64	
	48
37	37
30	
	23
	12
	12
	8
	7
	6
	0

a. Draw an ROC curve for this test.



(Answer: see below.)



(End Answer).

b. Estimate the area under the ROC curve. (Hint: Count boxes and divide by 5 ×10 = 50.)

0.87 (43.5 boxes). Shortcut: count boxes above the curve and subtract from 50, then divide by 50!

c. Now assign ranks to each distinct result. The highest result gets Rank = 1. Assign the average rank to ties. For example, if the same result appears twice after the result ranked 5, assign both occurrences the average rank (6 + 7)/2 = 6.5. If it occurs 5 times, assign all 5 occurrences rank 8 (the average of 6, 7, 8, 9, and 10). You can write the ranks next to the values in the sorted list above. (Hint: you can check your answer by remembering that the sum of all of the ranks should = N × (N+1)/2, where N is the total number of subjects.)

Begin Answer: It is easiest to do the ranks in two columns, as shown below:

Rank	Septic Arthritis	Rank	No Septic Arthritis
1	128		
2	112		
		3	71

	4	64		
			5	48
	6.5	37	6.5	37
	8	30		
			9	23
			10.5	12
			10.5	12
			12	8
			13	7
			14	6
			15	0
Totals	***21.5***		***98.5***	

End Answer

d. Now calculate the RANK SUM, S, as well as Smin and Smax.

S = 21.5 (sum of ranks in septic arthritis group)

Smin = ***d(d+1) / 2 = (5)(6) / 2 = 15***

Smax = ***dn + Smin = 50 + 15 = 65***

e. Now use the formula given in Chapter 3 to determine the area under the ROC curve from these ranks. You should get the same answer you got for part b above. Isn't that satisfying?

c = (Smax - S) / dn = (65 – 21.5) / 50 = 0.87

For math/puzzle nerds only:

f. If we measured WBC count out to enough significant digits, we could eliminate ties. If there are no ties, how many different possible ROC curves are there? This is the number of different 15-step paths from the lower left-hand corner to the upper right-hand corner, when the only allowed steps are up one or over one (to the right).

Answer: 15 choose 5 = 3003

g. Again assuming no ties, how many different ROC Curve areas are there?

Answer: 51. 0, 1, 2, 3, …, 50 boxes.